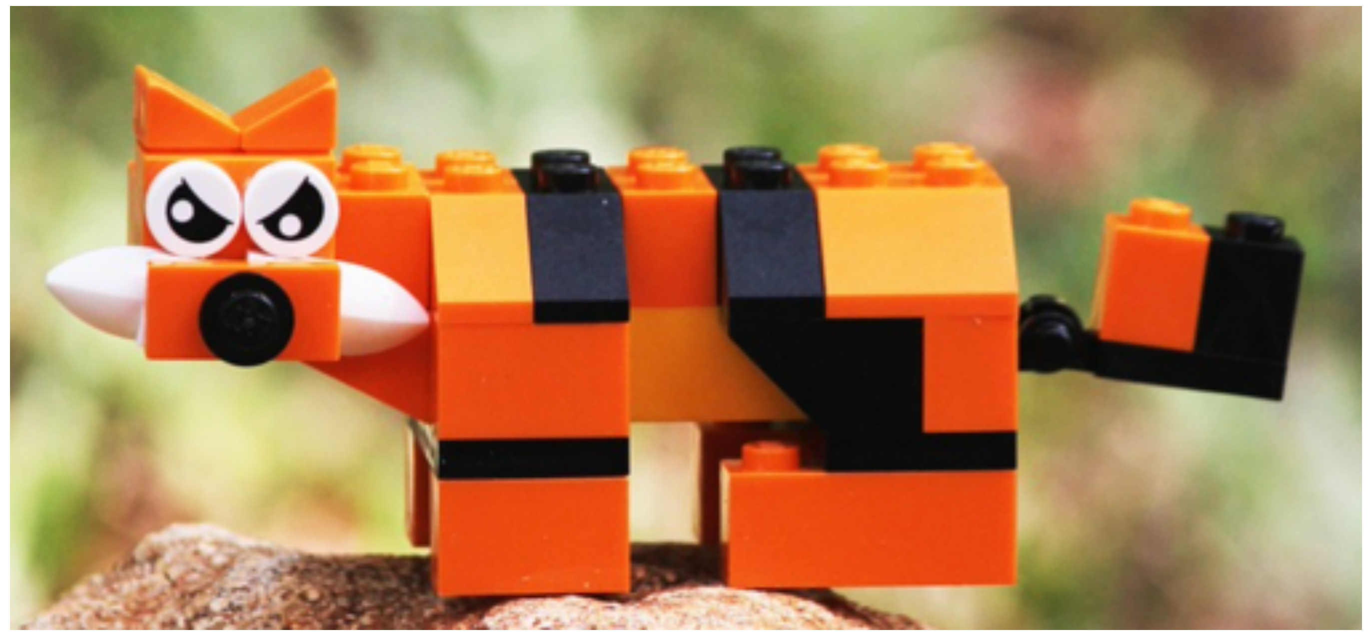
Modeling in CP

Bin-Packing Case Study

Modeling is an Art

- Modeling a real world problem with variables, domains and constraints

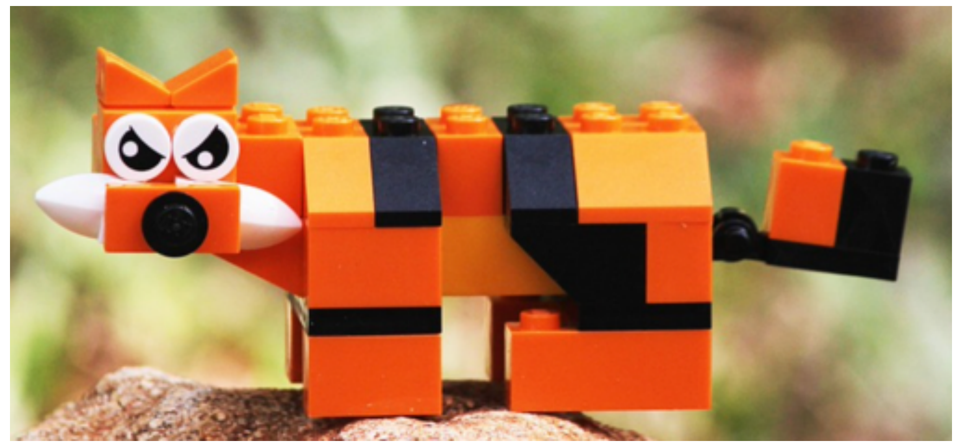
Model



Real world problem

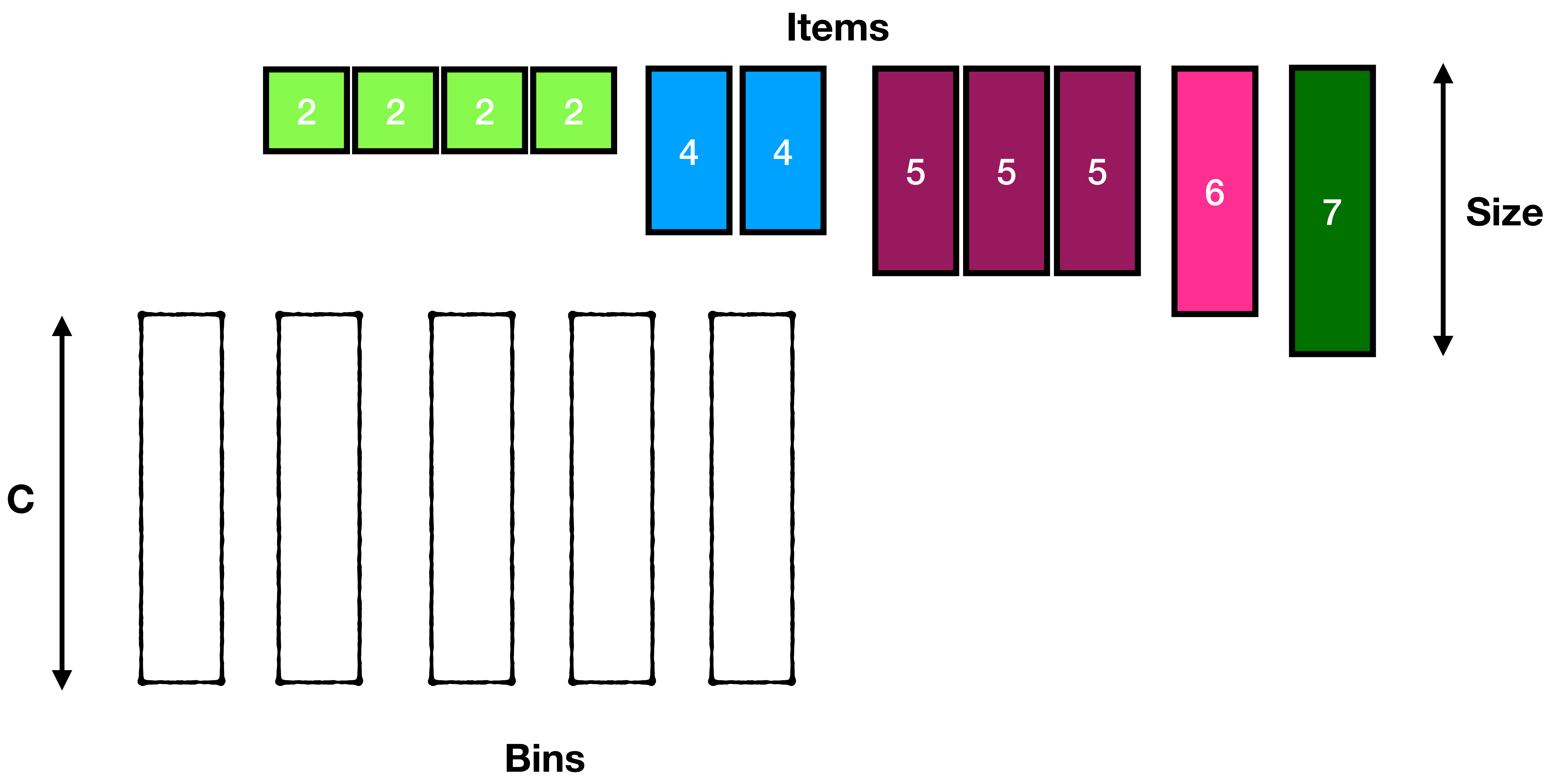
Modeling is an Art

- For a same problem, many different models
  - Variables, domains and constraints
- The model can have dramatic effect on the solving time

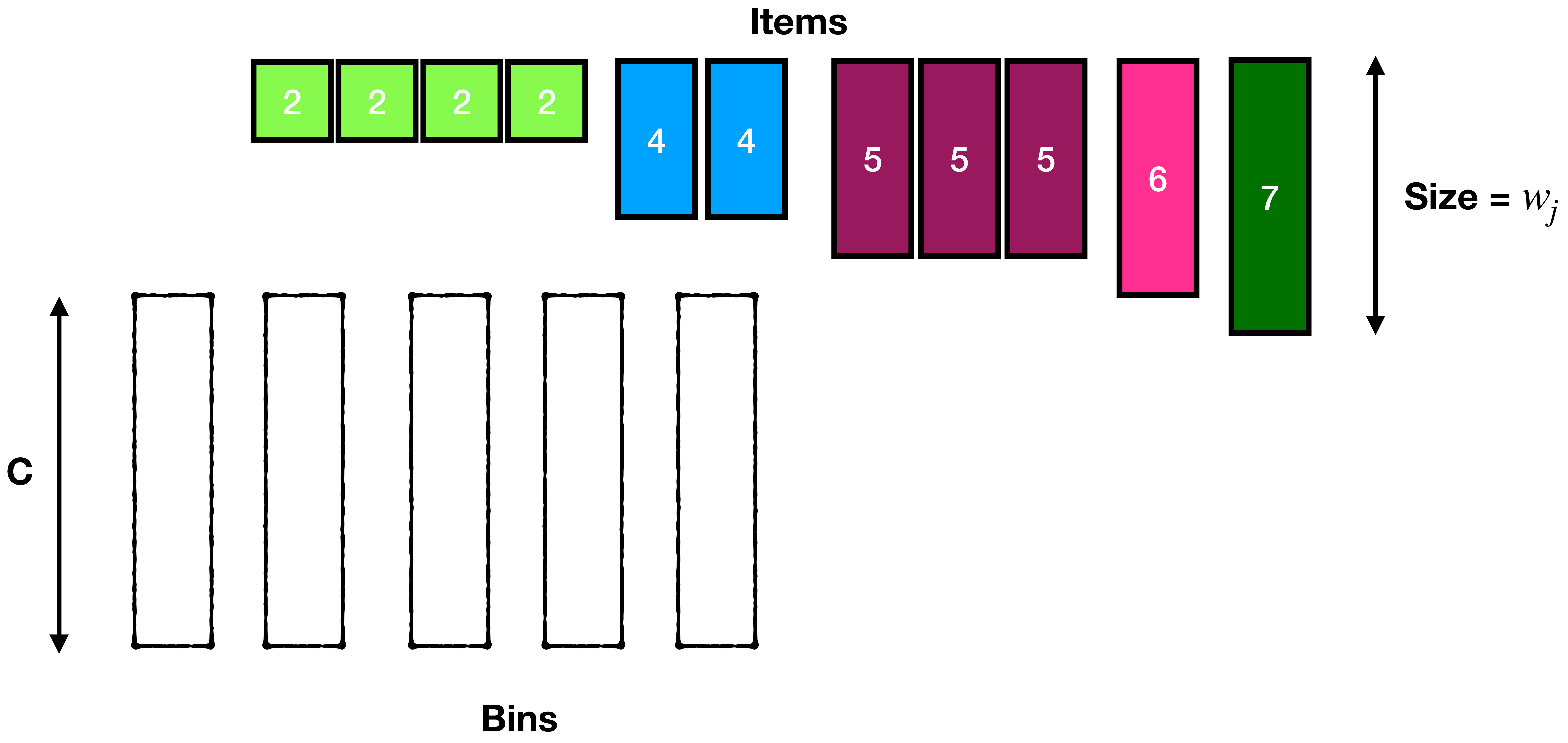

Case Study: Bin Packing

- Given n items, the size of each item
- Given m bins, each with a same capacity c
- Find a bin for each object such that the capacity of the bins is respected



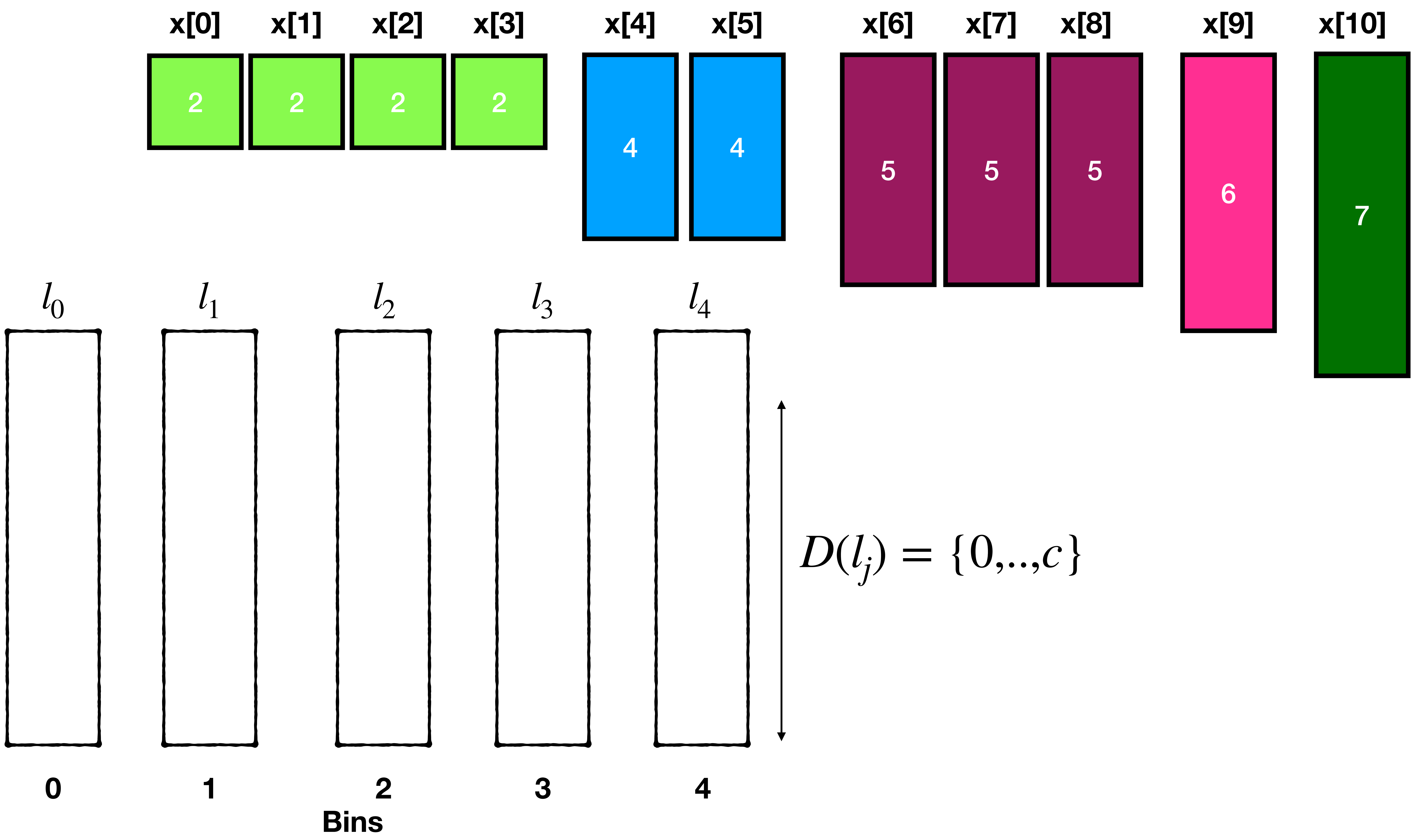
Decision variable ?

- Item point of view: in what bin do we place each item
- Bin point of view: what are the set of items allocated to each bin (set variable, more complex)



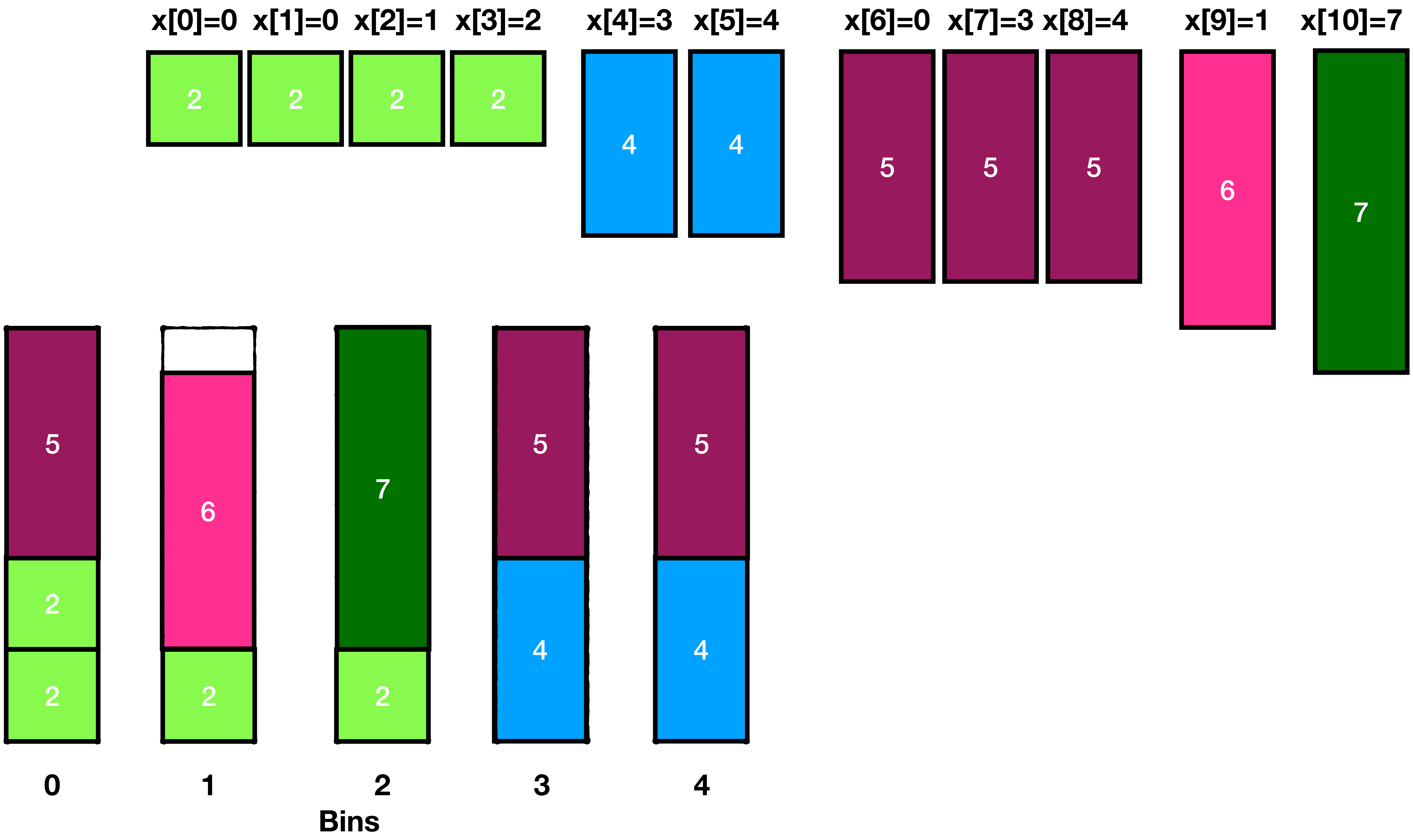
Decision variables and Domains

$D(x_i) = \{0,1,2,3,4\}$



$D(l_j) = \{0,..,c\}$

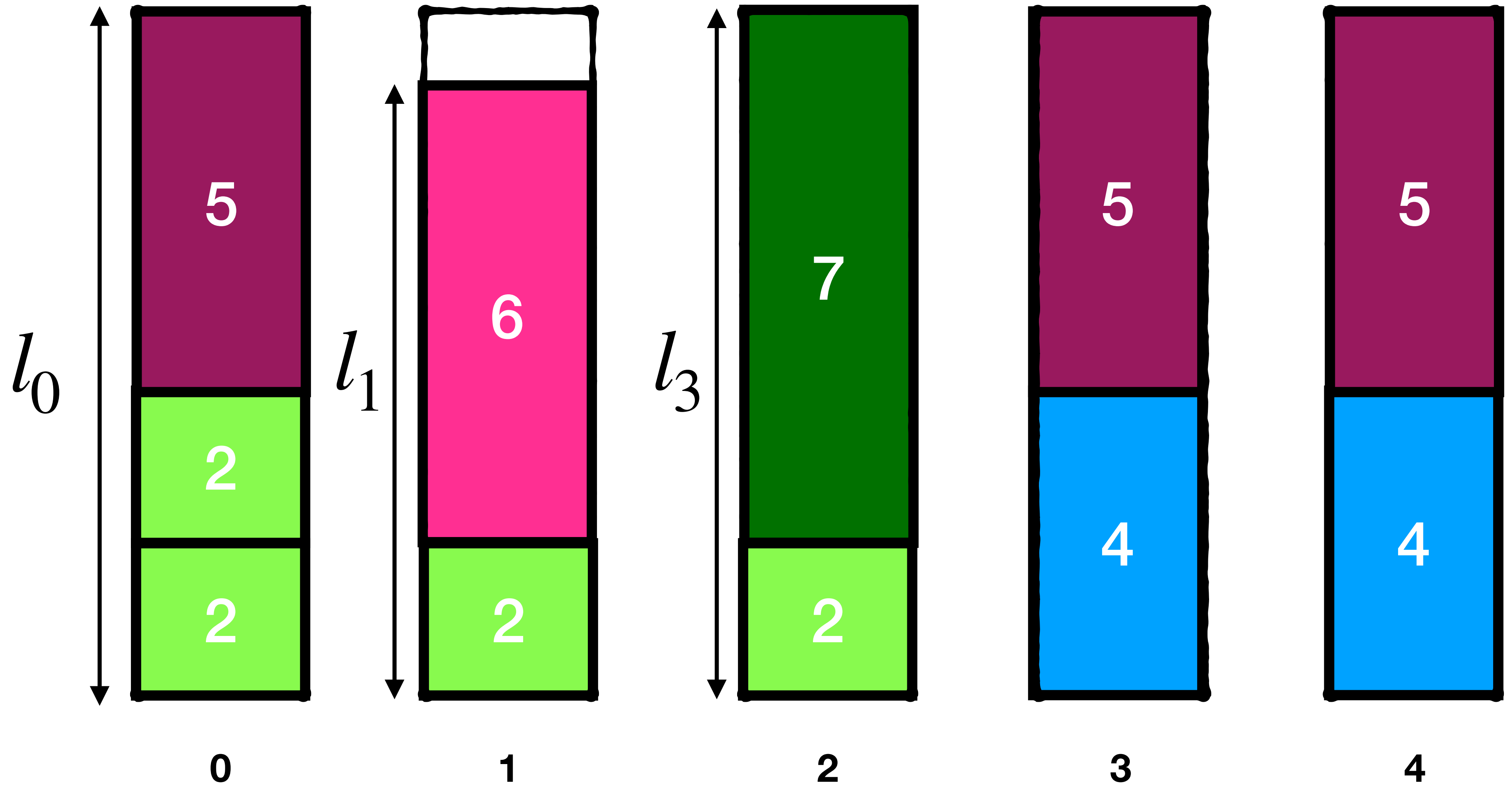
Solution


x[0]=0
x[1]=0
x[2]=1
x[3]=2
x[4]=3
x[5]=4
x[6]=0
x[7]=3
x[8]=4
x[9]=1
x[10]=7
2
2
2
2
4
4
5
5
5
6
7
5
2
2
6
2
7
2
5
4
5
4
0
1
2
3
4
Bins

Constraints

$$\forall j \in [1..m] : l_j = \sum_{i \in [1..n]} (x_i = j) \cdot w_i$$

$$D(l_j) = \{0,..,c\}$$



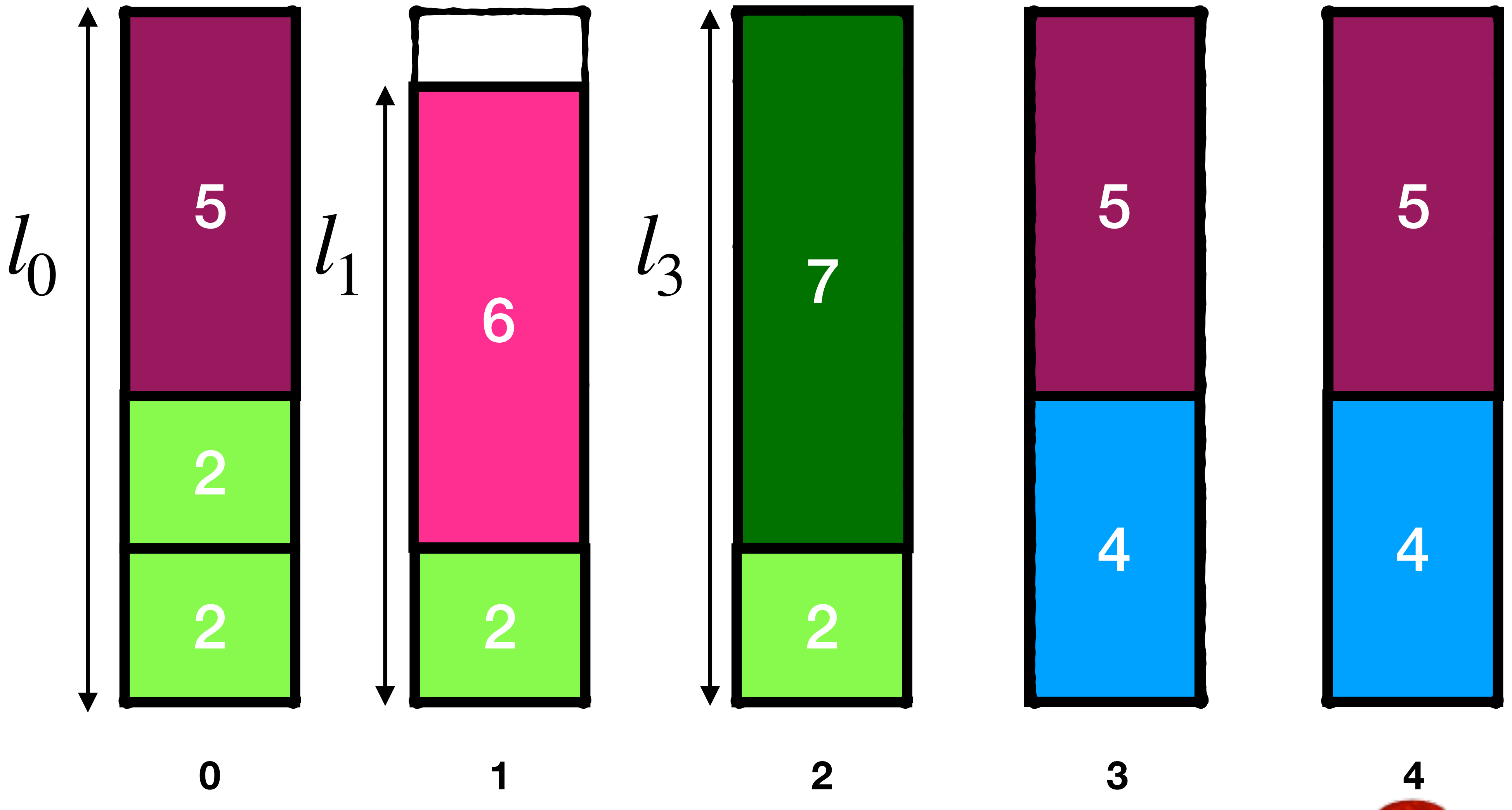
Bin-Packing Model

```
int capa = 9;
int [] items = new int[] {2,2,2,2,4,4,5,5,5,6,7};

int nBins = 5;
int nItems = items.length;

Solver cp = makeSolver();
IntVar []  x = makeIntVarArray(cp, nItems,nBins);
IntVar []  l = makeIntVarArray(cp, nBins,capa+1);

BoolVar [][] inBin = new BoolVar[nBins][nItems]; // inBin[j][i] = 1 if item i is placed in bin j
// bin packing constraint
for (int j = 0; j < nBins; j++) {
    for (int i = 0; i < nItems; i++) {
        inBin[j][i] = isEqual(x[i], j);
    }
}
for (int j = 0; j < nBins; j++) {
    IntVar[] wj = new IntVar[nItems];
    for (int i = 0; i < nItems; i++) {
        wj[i] = mul(inBin[j][i], items[i]);
    }
    cp.post(sum(wj, l[j]));
}

DFSearch dfs = makeDfs(cp,firstFail(x));
```

$$l_j = \sum_{i \in [1..n]} (x_i = j) \cdot w_i$$

Global Constraints for Bin-Packing

$$\forall j \in [1..m] : l_j = \sum_{i \in [1..n]} (x_i = j) \cdot w_i$$

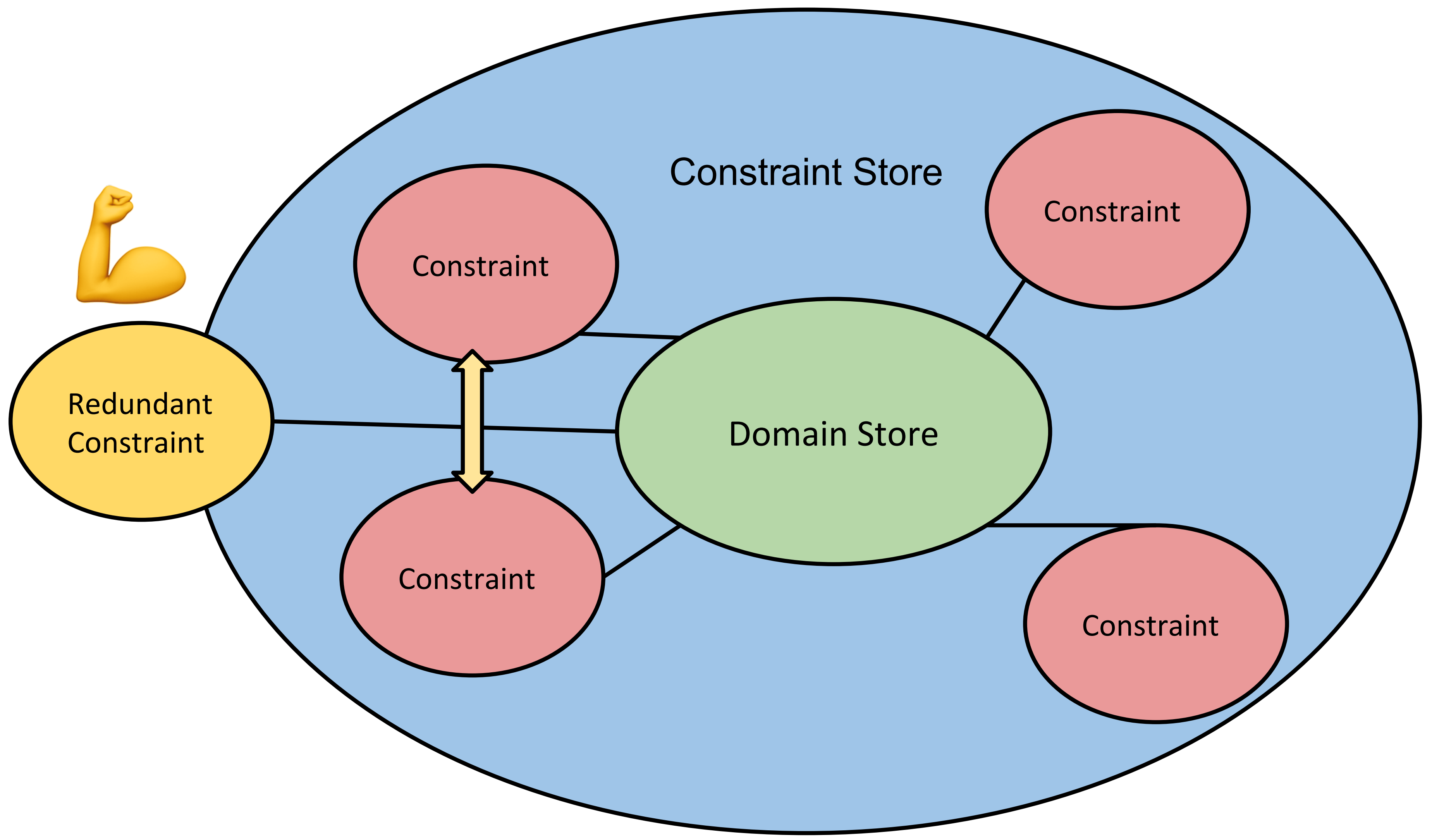


- This kind of constraint is very frequent, most of the solvers call it $\text{BinPacking}([l_1, \ldots, l_m], [x_1, \ldots, x_n], [w_1, \ldots, w_n])$

- Shaw, Paul. "A constraint for bin packing." CP 2004.
- Schaus, Pierre. "Solving balancing and bin-packing problems with constraint programming." *PhD Thesis* (2009)

Redundant Constraints

- Redundant Constraints:
  - Do not exclude any previous solution
  - Improve the pruning of the search space (better communication between constraints)



How to find redundant constraints for your model ?

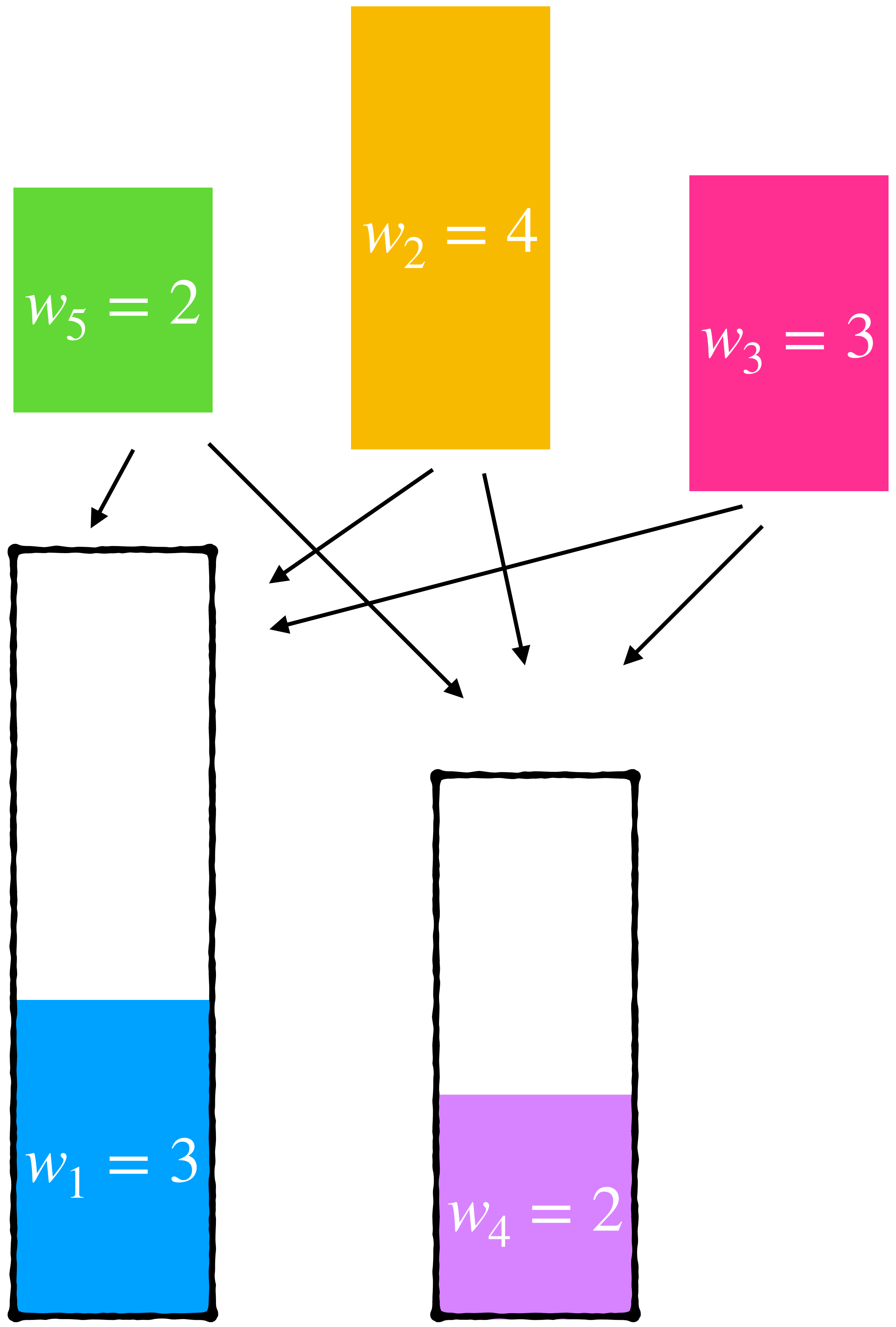
- Express properties of the solution
- Derive consequences of (combinations of) constraints
- BinPacking:

$$\forall j \in [1..m] : l_j = \sum_{i \in [1..n]} (x_i = j) \cdot w_i$$

$$\boxed{\sum_{j \in [1..m]} l_j = \sum_{i \in [1..n]} w_i}$$ 💪

Redundant Constraint

Infeasible, but not detected by $\forall j \in [1..m] : l_j = \sum_{i \in [1..n]} (x_i = j) \cdot w_i$



Failure detected by redundant constraints

$$\sum_{j \in [1..m]} l_j = \sum_{i \in [1..n]} w_i$$

$$[3..7] + [2..5] = 14$$

$$[5..12] = 14$$

Bin-Packing Model

```java
int capa = 9;
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int nBins = 5;
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Solver cp = makeSolver();
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BoolVar [][] inBin = new BoolVar[nBins][nItems]; // inBin[j][i] = 1 if item i is placed in bin j
// bin packing constraint
for (int j = 0; j < nBins; j++) {
    for (int i = 0; i < nItems; i++) {
        inBin[j][i] = isEqual(x[i], j);
    }
}
for (int j = 0; j < nBins; j++) {
    IntVar[] wj = new IntVar[nItems];
    for (int i = 0; i < nItems; i++) {
        wj[i] = mul(inBin[j][i], items[i]);
    }
    cp.post(sum(wj, l[j]));
}

// redundant constraint 💪
cp.post(sum(l, IntStream.of(items).sum()));
```
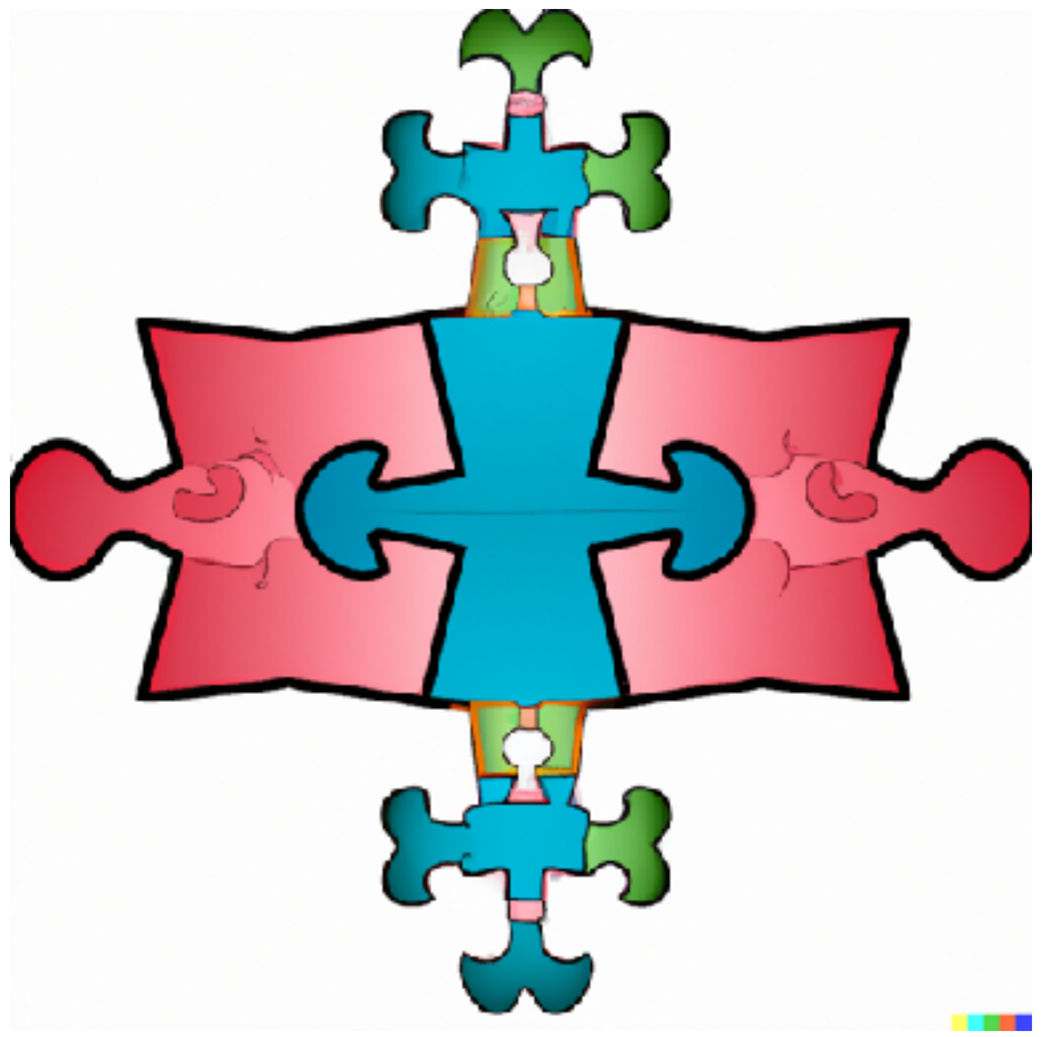
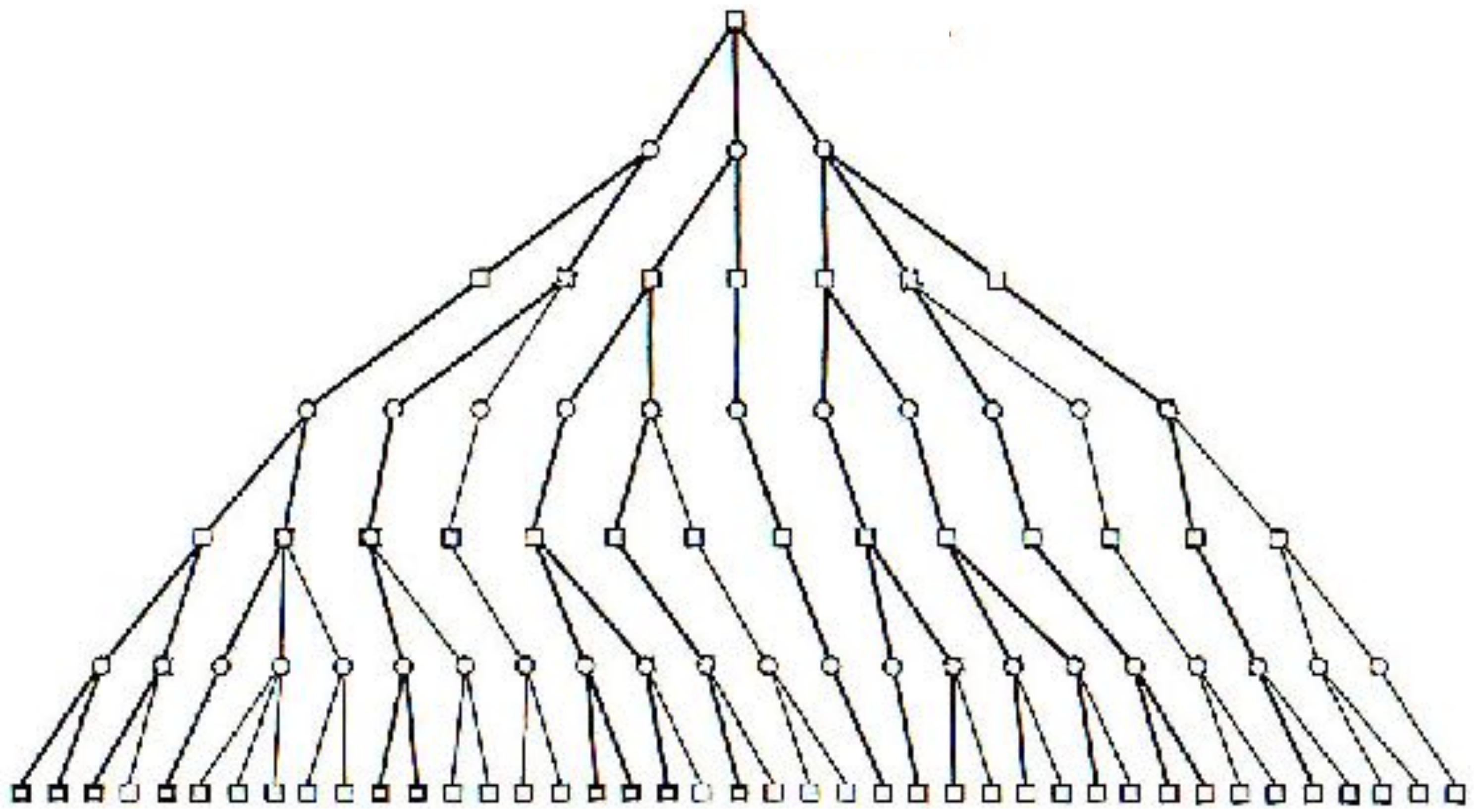

Bin-Packing

Symmetries



Symmetries

- Many problems naturally exhibit symmetries
- A symmetry maps solutions to solutions and non-solutions to non-solutions
- Symmetries leads to symmetrical search spaces
- Exploring symmetrical search spaces is useless
  - If no solution in one, no solution in the other



Detect and remove symmetries (dynamic or static)

only inspect one (non-)solution in each equivalence class

Value Symmetry

A value symmetry is a bijection σ on values mapping (non-)solutions to (non-)solutions:

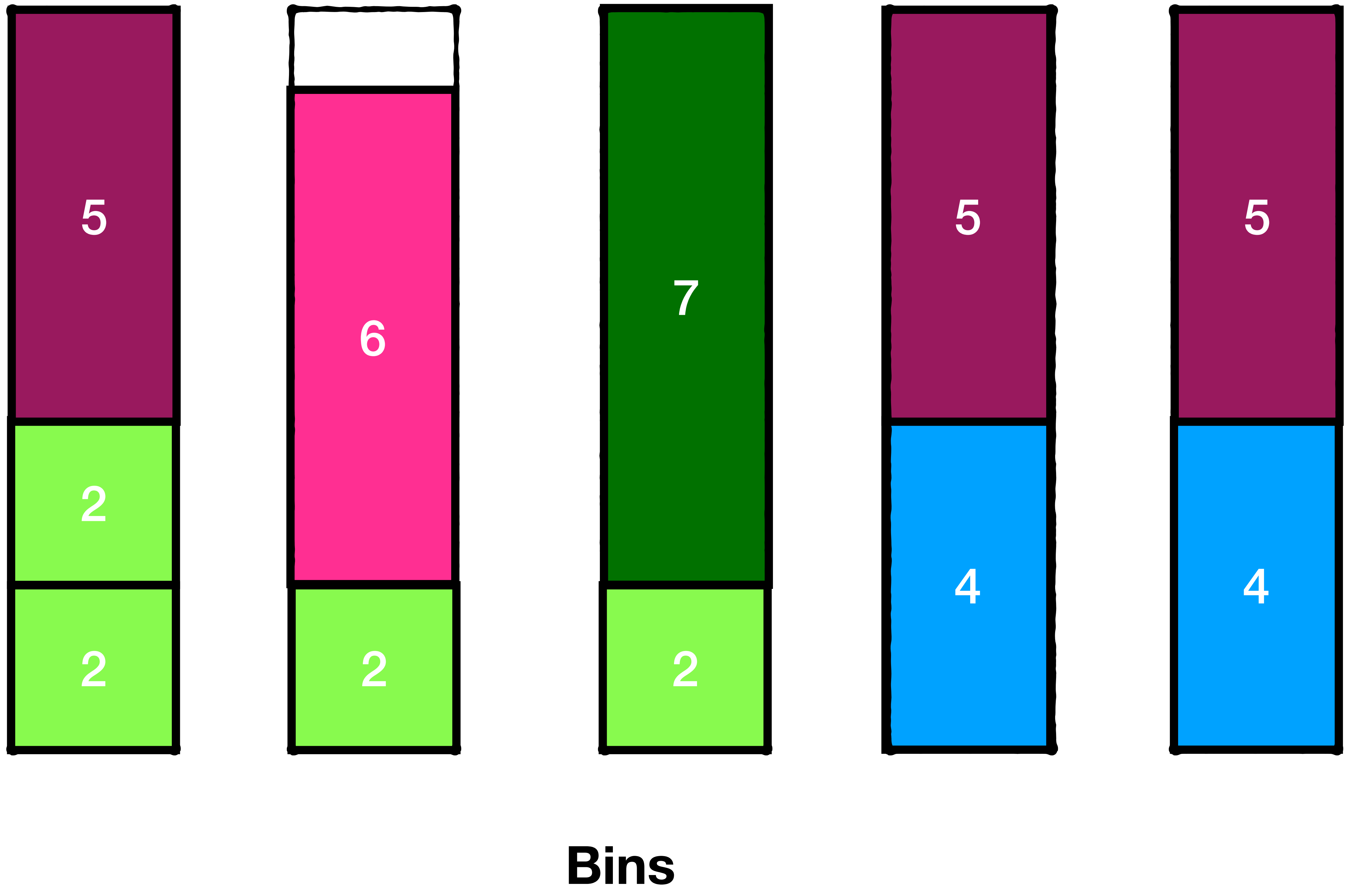
$$x_1, x_2, \ldots, x_n \qquad \overset{\sigma}{\longleftrightarrow} \qquad x_{\sigma(1)}, x_{\sigma(2)}, \ldots, x_{\sigma(n)}$$

$$a_1, a_2, \ldots, a_n \qquad \qquad \sigma(a_1), \sigma(a_2), \ldots, \sigma(a_n)$$

`Value symmetries change the values`

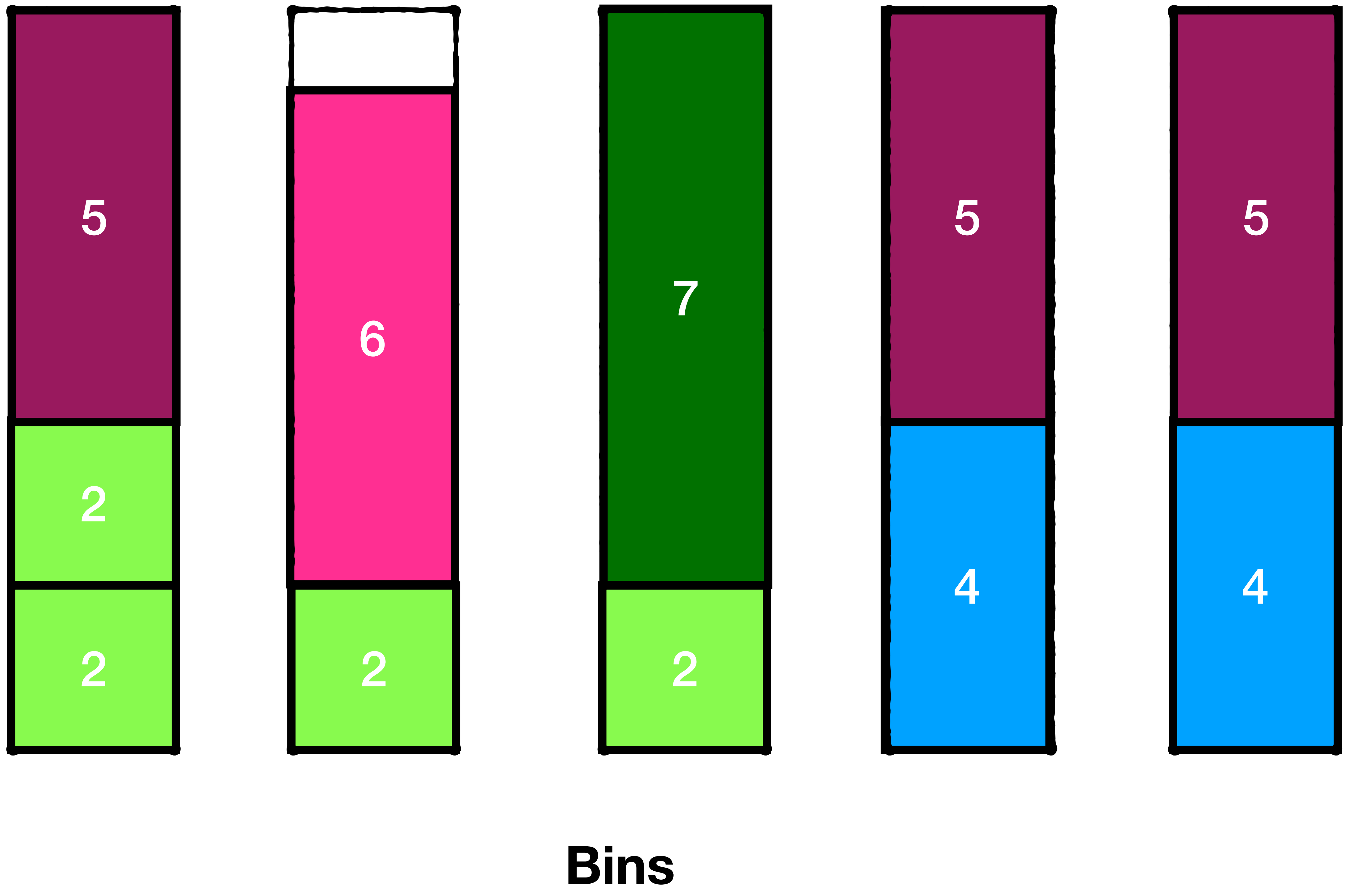
Bin-Packing (value) Symmetries

- Interchanging bins is still a valid solution



Value symmetry breaking

- Solution: Impose an order on the bins
- For example: increasing loads $l[0] \leq l[1] \leq l[2] \leq l[3] \leq l[4]$
- Does not remove all symmetries in case of ties



Better: Lexicographic Constraints

- Impose a total order on bins (no ties)

```
BoolVar [][] inBin = new BoolVar[nBins][nItems]; // inBin[j][i] = 1 if item i is placed in bin j
// bin packing constraint
for (int j = 0; j < nBins; j++) {
    for (int i = 0; i < nItems; i++) {
        inBin[j][i] = isEqual(x[i], j);
    }
}
for (int j = 0; j < nBins-1; j++) {
    cp.post( inBin[j] ≼ inBin[j+1]);
}
```

Lexicographic Ordering

Frisch, A., Hnich, B., Kiziltan, Z., Miguel, I., & Walsh, T. (2002, September). Global constraints for lexicographic orderings. CP2002

Variable Symmetry

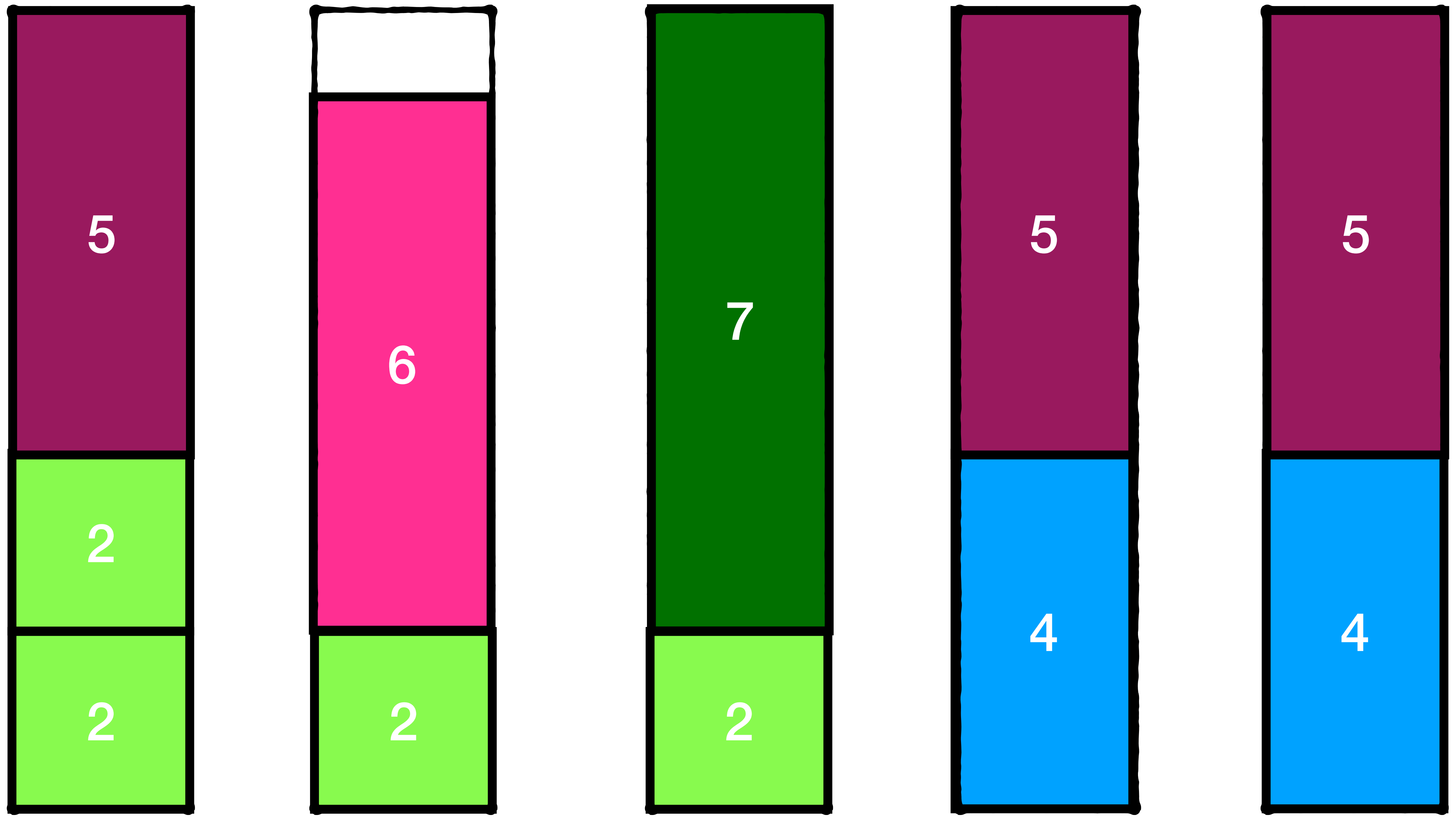
A variable symmetry is a bijection σ on variables mapping (non-)solutions to (non-)solutions:

$$x_1, x_2, \ldots, x_n \quad \overset{\sigma}{\longleftrightarrow} \quad x_{\sigma(1)}, x_{\sigma(2)}, \ldots, x_{\sigma(n)}$$
$$a_1, a_2, \ldots, a_n \qquad\qquad a_1, a_2, \ldots, a_n$$

`Variable symmetries swap the variables`

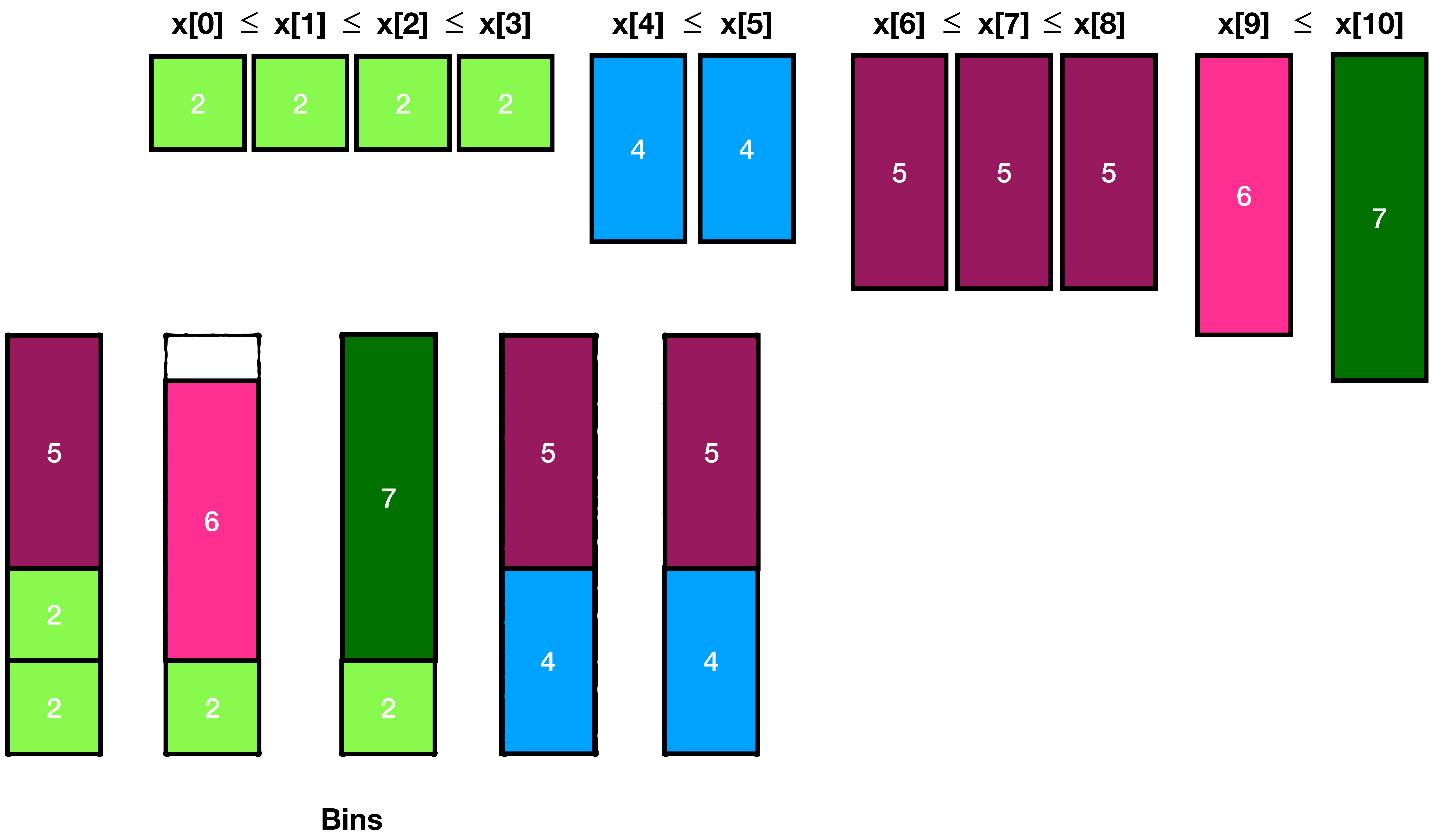
Bin-Packing (variable) Symmetries

- Exchanging similar items



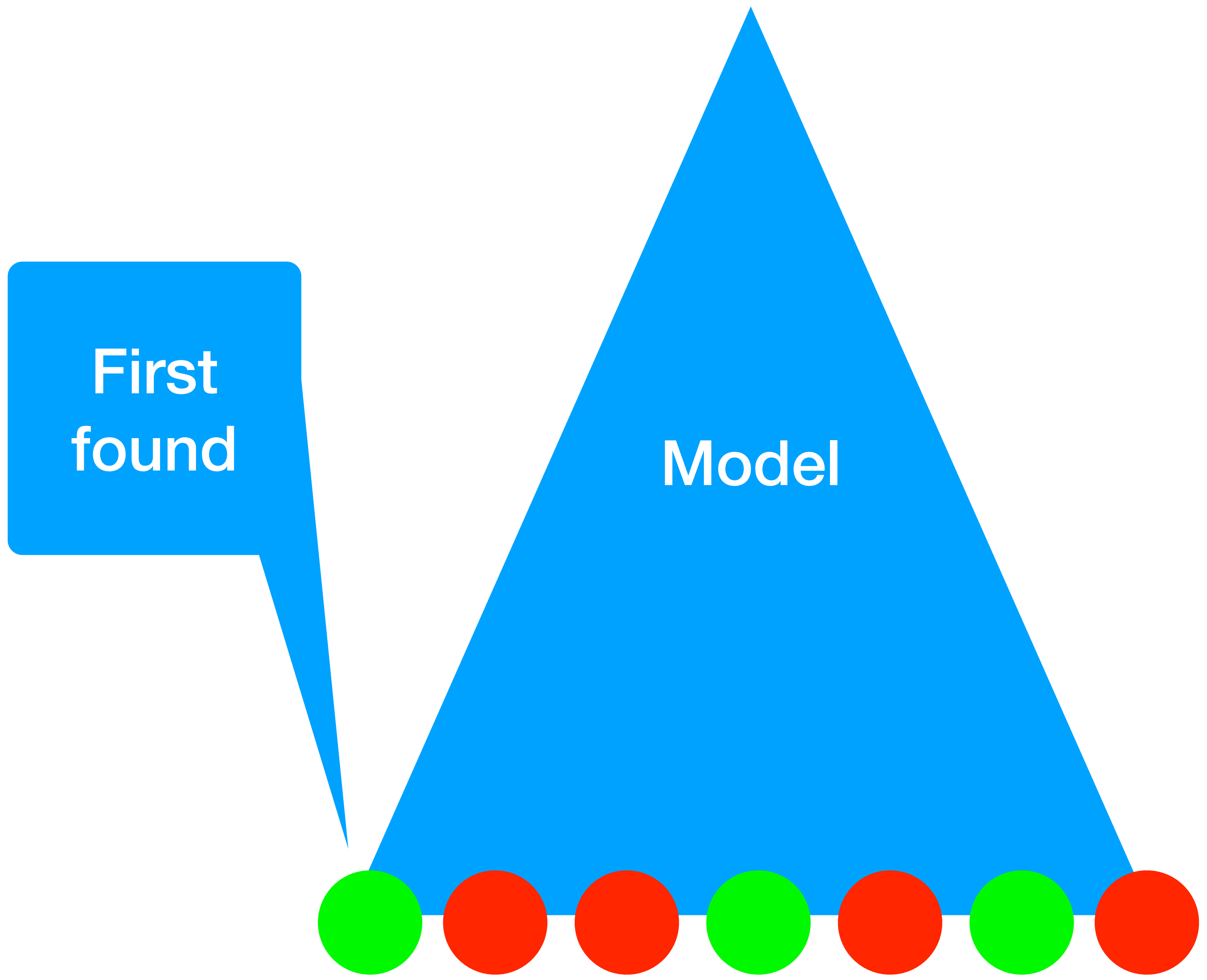
Bins

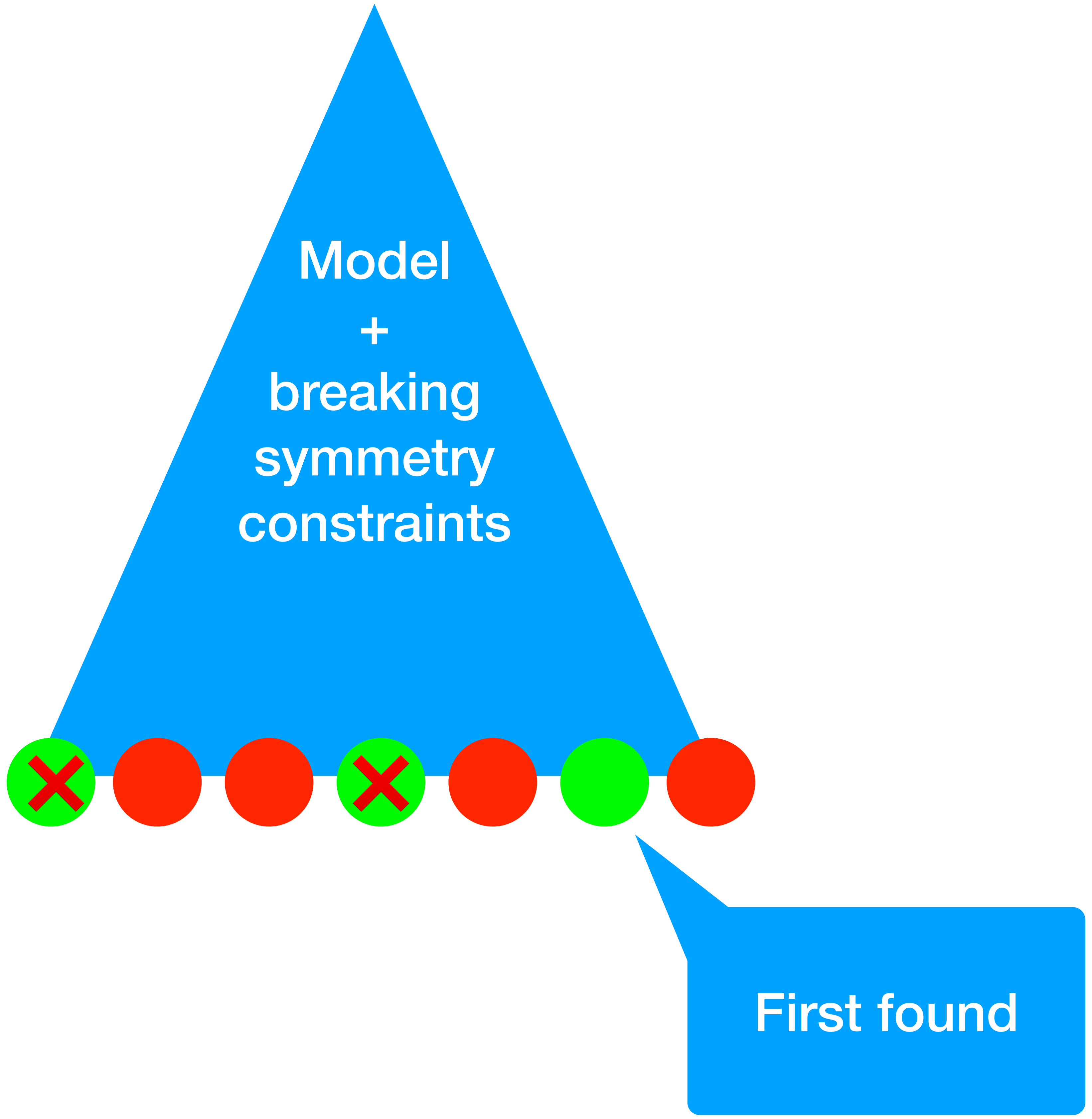
Bin-Packing: Breaking variable symmetries



Drawback of symmetry breaking with constraints

- ⚠️ Sometimes useful, sometimes not
- Be careful because you suppress solutions.
- Consequence:
  - Solution discovered very early in the search tree might not exist anymore (bad interaction with the heuristic).



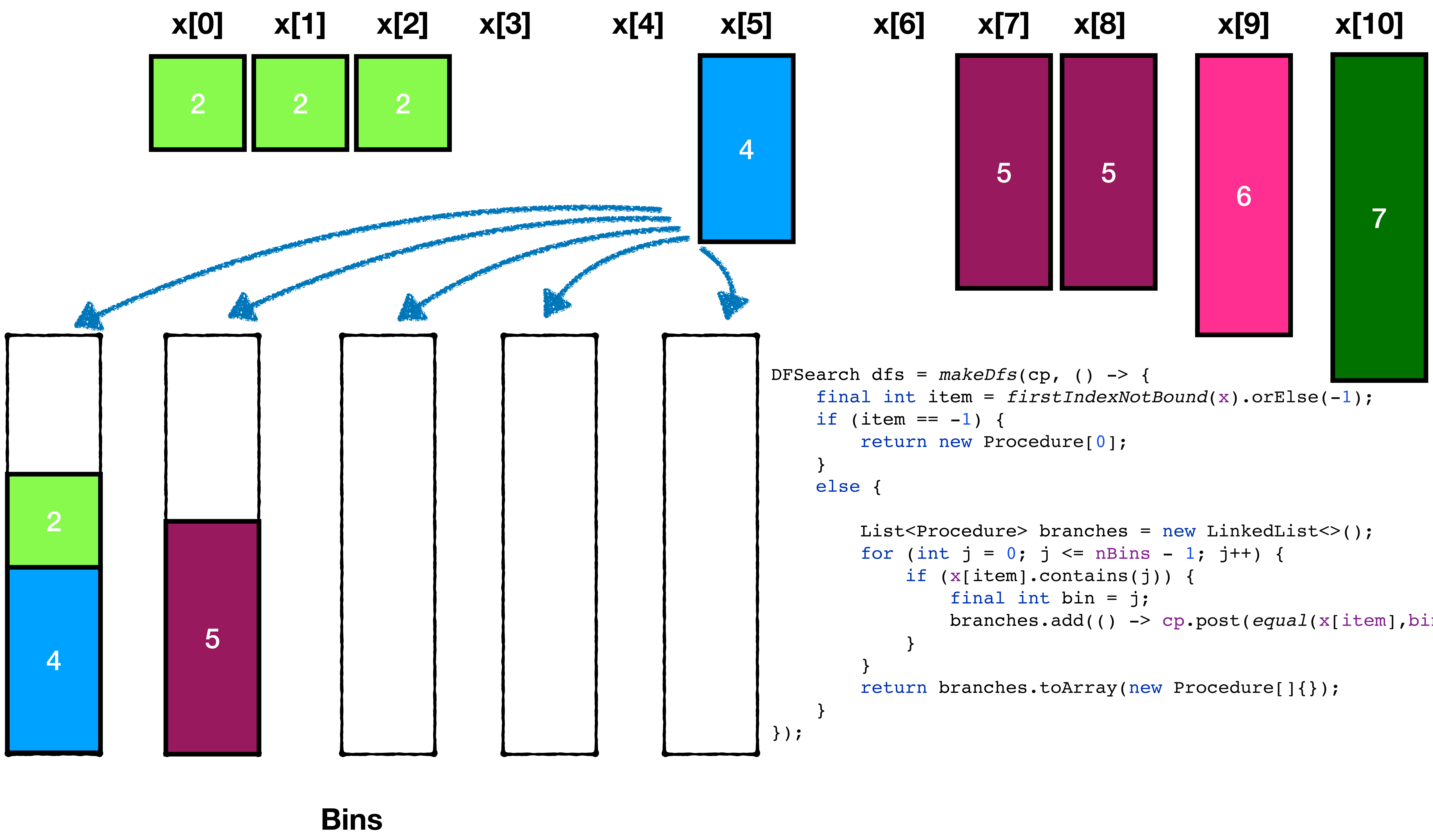


Challenge

- Is it possible to remove variable/value symmetries such that the first solution remains the same ?

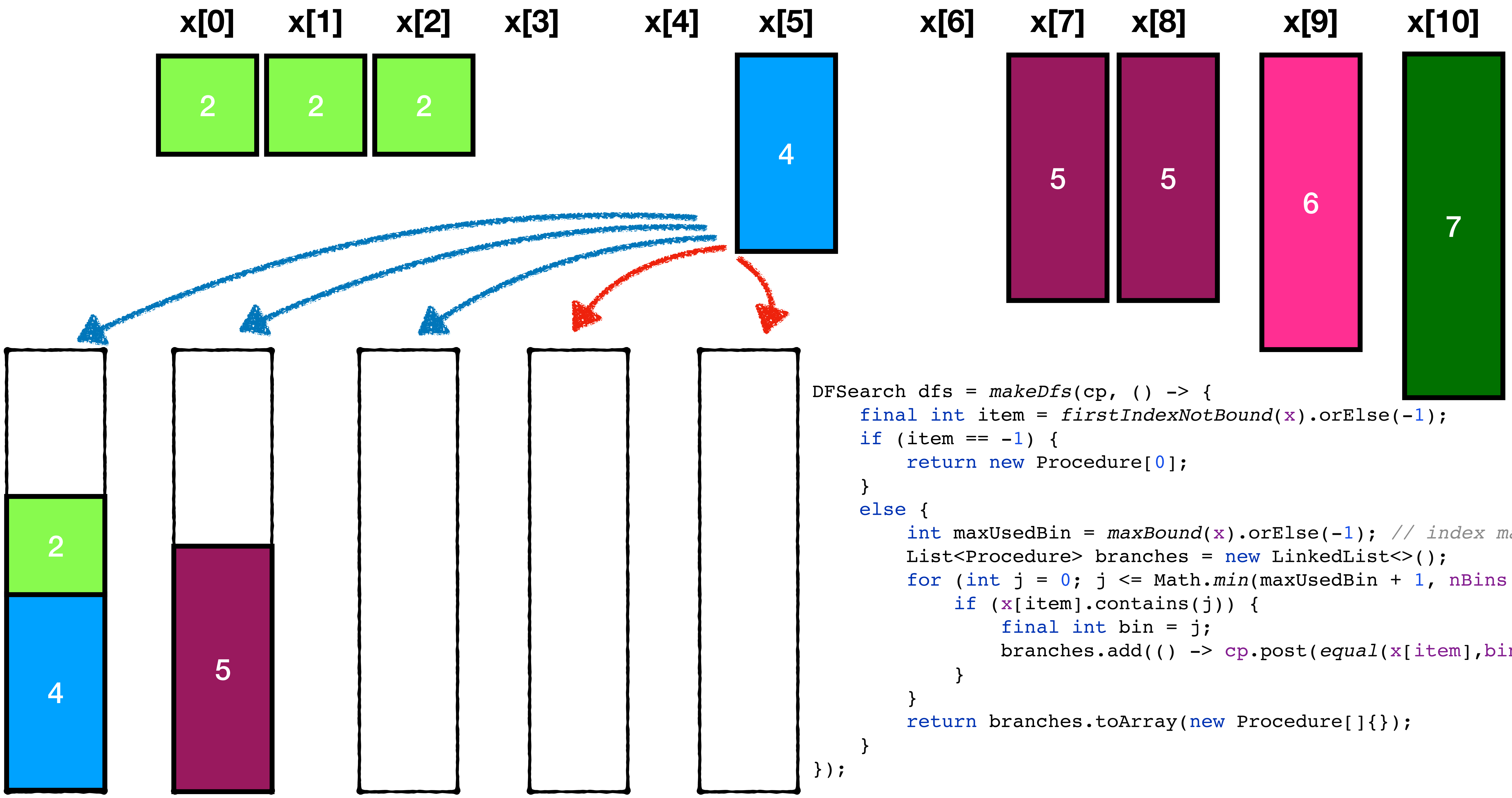
- Yes! Dynamic symmetry breaking = Add constraints during search
  - each time a (non-)solution is found)
  - Special search heuristic

Dynamic Symmetry Breaking during Search




```
DFSearch dfs = makeDfs(cp, () -> {
    final int item = firstIndexNotBound(x).orElse(-1);
    if (item == -1) {
        return new Procedure[0];
    }
    else {

        List<Procedure> branches = new LinkedList<>();
        for (int j = 0; j <= nBins - 1; j++) {
            if (x[item].contains(j)) {
                final int bin = j;
                branches.add(() -> cp.post(equal(x[item],bin)));
            }
        }
        return branches.toArray(new Procedure[]{});
    }
});
```

Dynamic Symmetry Breaking during Search




```
DFSearch dfs = makeDfs(cp, () -> {
    final int item = firstIndexNotBound(x).orElse(-1);
    if (item == -1) {
        return new Procedure[0];
    }
    else {
        int maxUsedBin = maxBound(x).orElse(-1); // index max used bin
        List<Procedure> branches = new LinkedList<>();
        for (int j = 0; j <= Math.min(maxUsedBin + 1, nBins - 1); j++) {
            if (x[item].contains(j)) {
                final int bin = j;
                branches.add(() -> cp.post(equal(x[item],bin)));
            }
        }
        return branches.toArray(new Procedure[]{});
    }
});
```

Bins

Symmetry breaking

- Static Symmetry Breaking
  - Use different variables
  - Add constraints to the model (ex: lexicographic)
- Dynamic Symmetry breaking (during search)
  - Add constraints during the search (each time a (non—solution is found)
  - Use special search heuristics

- Breaking symmetries does not always help (symmetries removed but so might be the left-most solution)

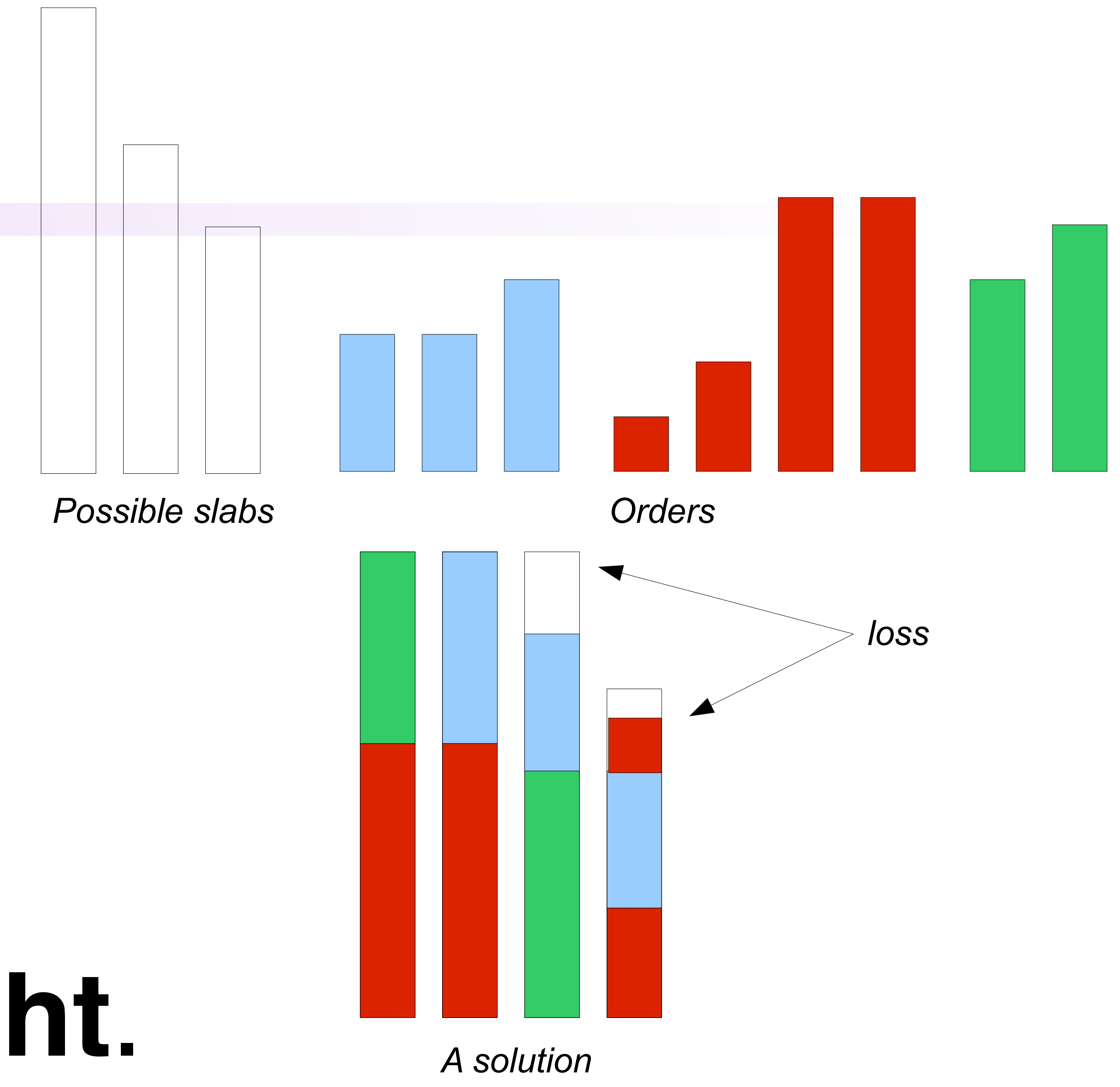
Bin-Packing Demo … 👩‍💻 👨‍💻

Steel Mill Slab Problem

(Programming Assignment)

https://www.csplib.org/Problems/prob038/

The Steel Mill Slab Problem



- Steel produced by casting molten iron into slabs.
- Only a finite number of slab sizes.
- An order has two properties,
  - a **color** (route required through the steel mill) and + **weight**.
- Given n input orders, assign the orders to slabs, the number and size of which are also to be determined, such that **the total weight of steel produced is minimized**.
- Assignment subject to constraints:
  - Capacity: The total weight assigned to a slab cannot exceed the slab capacity.
  - Colors: Each slab can contain at most 2 colors.

Notations

- n is the number of orders
- $c_i \in \text{colors}$ is the color of order i
- $w_i \in \mathbb{N}^+$ is the weight of order i
- σ is the set of different slab capacity. At most m slabs will be used, we label them from 1 to m ($m = n$ if not restricted)

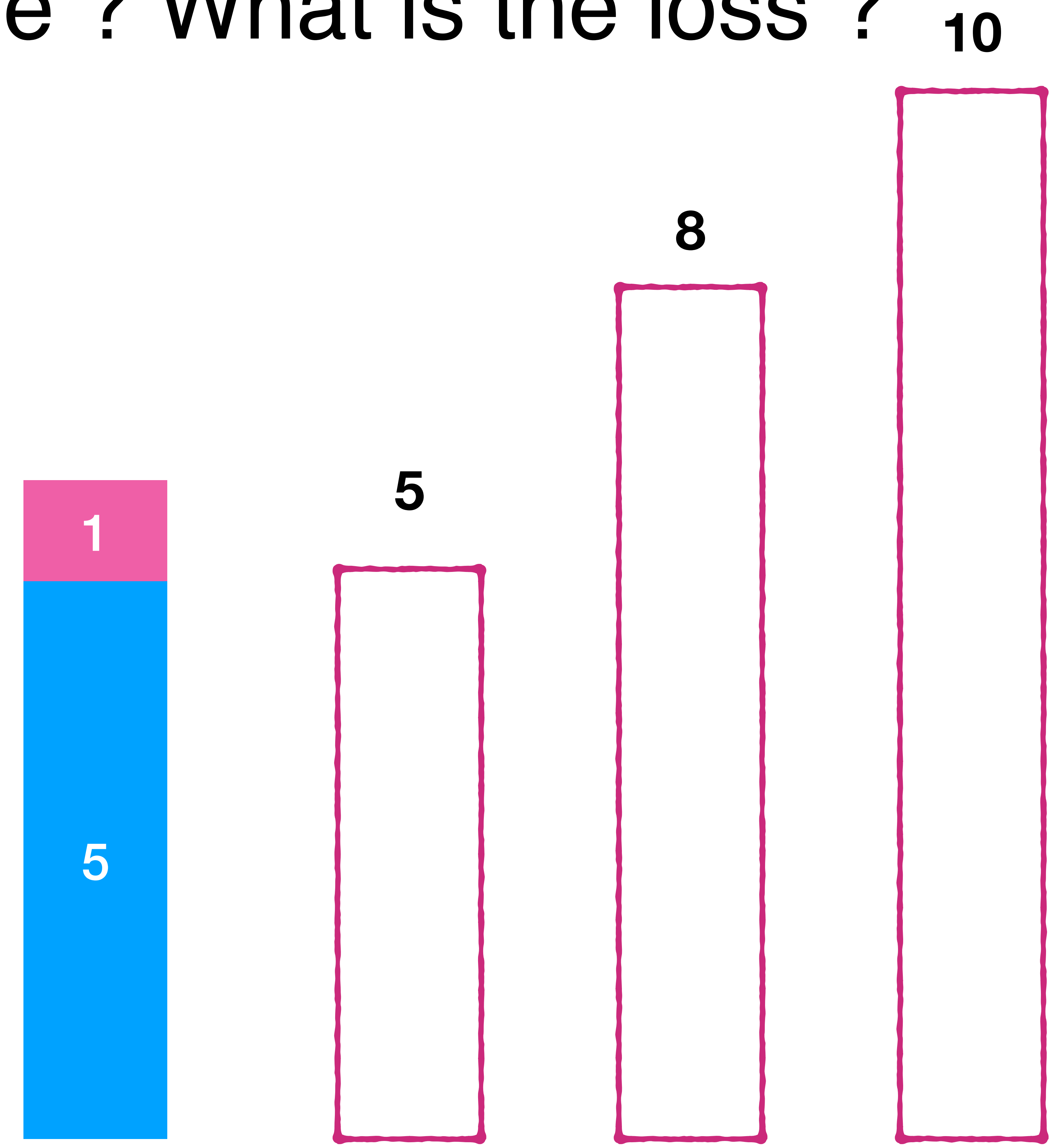
Model

- Decision variables:
  - $o_i \in [1..n]$ is the slab attributed to order i
- Auxiliary variables:
  - $p_j \in [0..\text{maxcapa}]$ is the weight of the orders attributed to slab j
  - $l_j \in [0..\text{maxcapa}]$ is the minimal loss of slab j (determined by the slab of minimal capacity $\geq p_j$)
- Objective

  minimize the total loss: $$\sum_{j \in [1..m]} l_j$$

Computing losses with Element Constraints

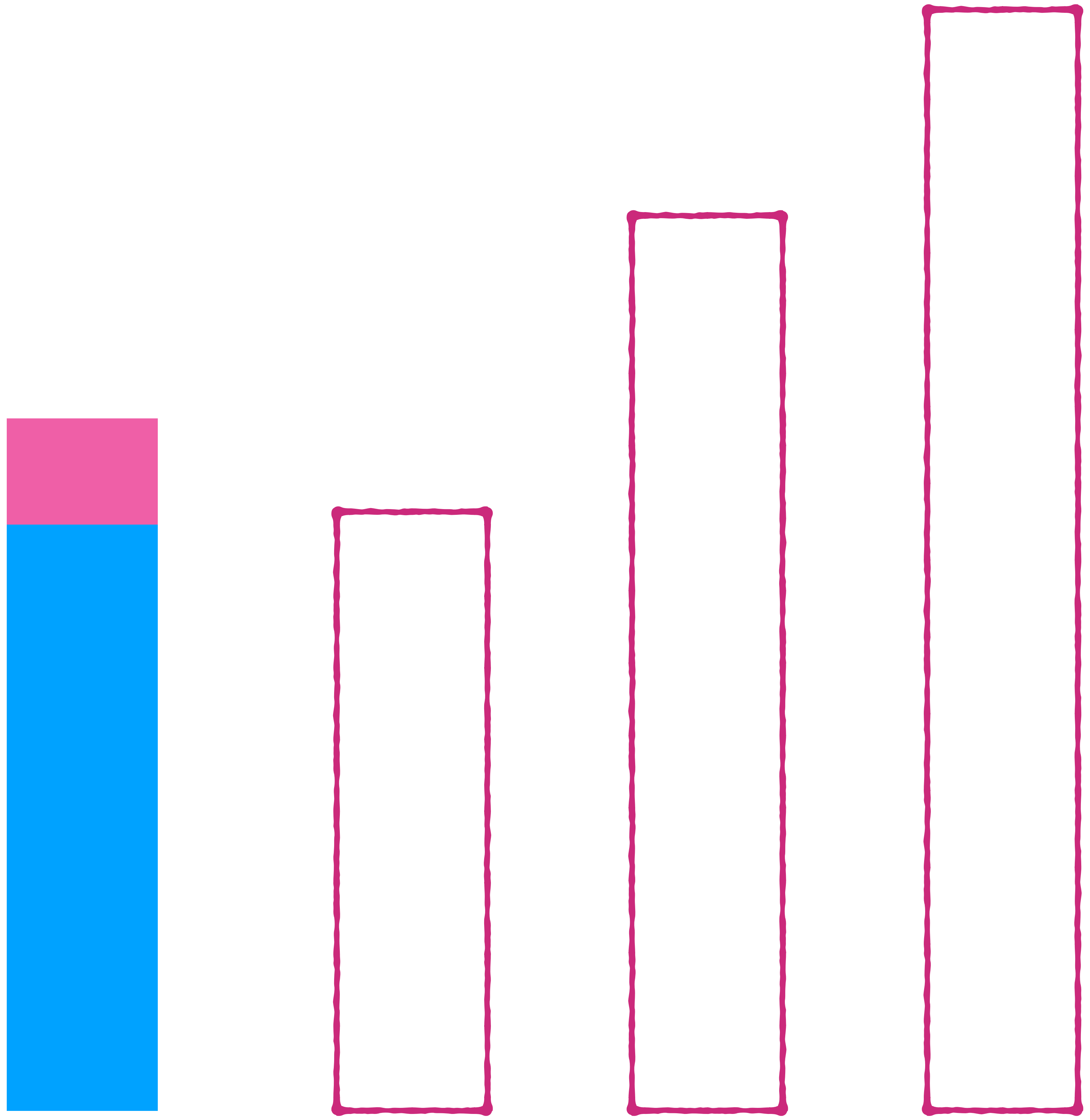
- Assume 3 slab capacities {5,8,10}, an order of size 5 and 1
- What slab to chose ? What is the loss ?



Computing losses with Element Constraints

- 💡 precompute the loss for every possible load

Load	Loss
10	0
9	1
8	0
7	1
6	2
5	0
4	1
3	2
2	3
1	4
0	0



Computing losses with Element Constraints

- Assume 3 slab capacities {5,8,10}
- We can preprocess an array L = [0,4,3,2,1,0,2,1,0,1,0].
- Loss for a total weight $p_j = 3$? L = [0,4,3,2,1,0,2,1,0,1,0].
- 💡 Use element constraints to link loss and weight variables: $l_j = L[p_j]$

element

Computing loads (Bin-Packing or Pack)

$$\forall j \in [1..m] : p_j = \sum_{i \in [1..n]} (o_i = j) \cdot w_i$$

At most two colors!

Logical Or Constraint (and watched literals)

Modeling the Color Constraints

- At most 2 colors / slab

  - Is color k used in slab j (yes 1, no 0)?

isEqual

isOr constraint
b iff (x_1 or x_2 or … or x_n)

$$\bigvee_{i \in [1..n] | c_i = k} (o_i = j)$$

  - $\forall j \in [1..n]$:

$$\sum_{k \in colors} \left(\bigvee_{i \in [1..n] | c_i = k} (o_i = j) \right) \leq 2.$$

isOr

Reified Or: IsOr Constraint

- b iff (x_1 or x_2 or … or x_n)
  - b = true: post (x_1 or x_2 or … or x_n) = the Or constraint and deactivate
  - b = false: set all variables x_i to false
  - x_i become true: set b to true and deactivate (we must listen to all variables)
  - all x_i's are false: set b to false (maintain them in a sparse-set)

The Or or Clause Constraint

- At least one boolean variable is true:
  - x_1 or x_2 or … or x_n

- Can only propagate when all variables are false, except one (this is called **unit propagation**).

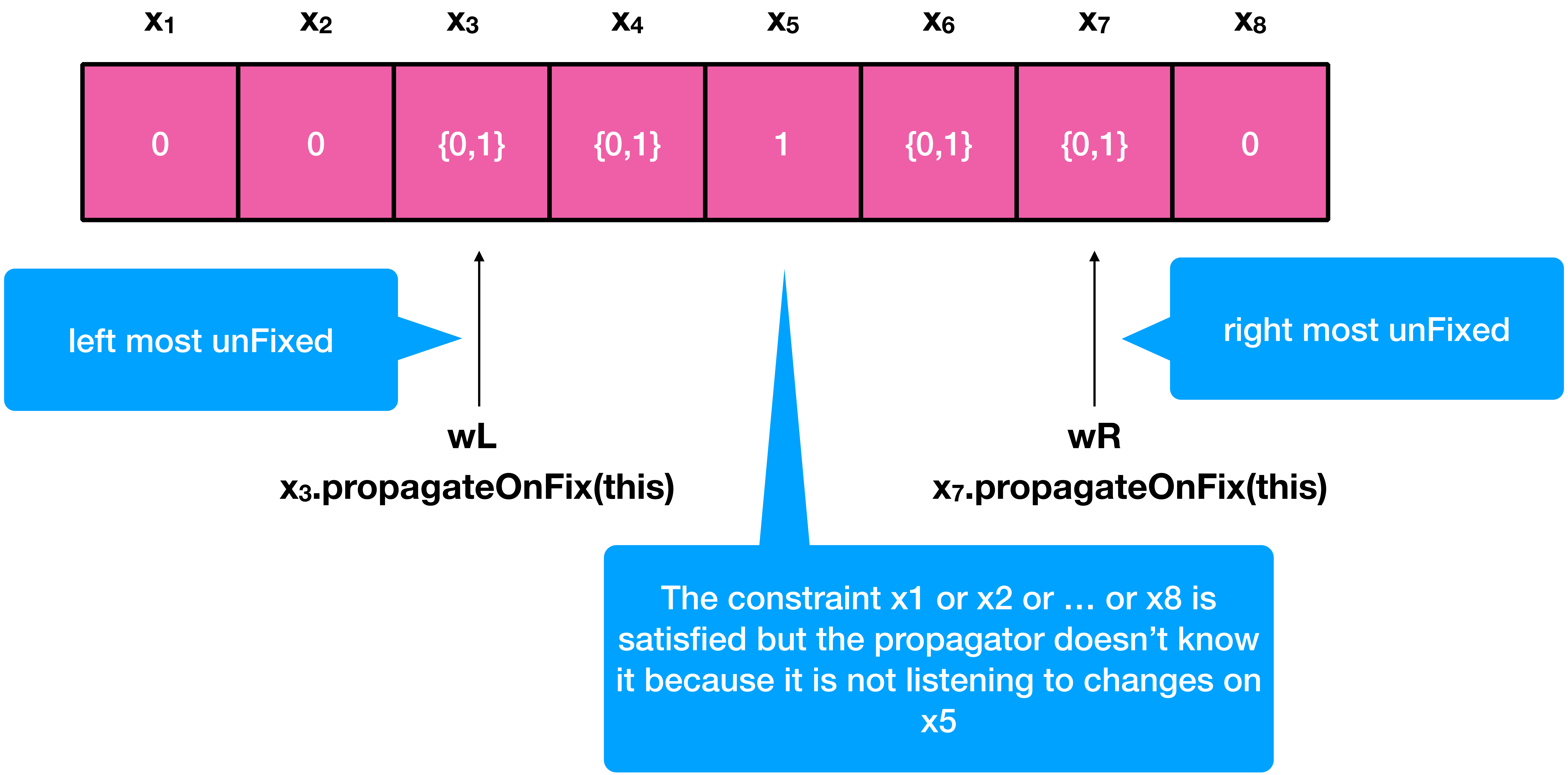
- This is the only propagation used in modern SAT solvers.

First implementation

- Listen to all variables
- Maintain sparse-set with unbound variables
- If one variable become true, deactivate the constraint because it is satisfied.
- If the sparse-set becomes empty and all other variables are false, throw an InconsistencyException
- If only one variable is unbound, the other ones are false, set the last one to true (unit propagation)
- Can be done with O(1) per variable change but can we do better?

Watched literal (adapted to MiniCP)

- Don't listen to all changes, only listening to two variables is enough.
- Idea: If two variables are either unassigned or assigned true, no need to do anything.



Watched literal (adapted to MiniCP)

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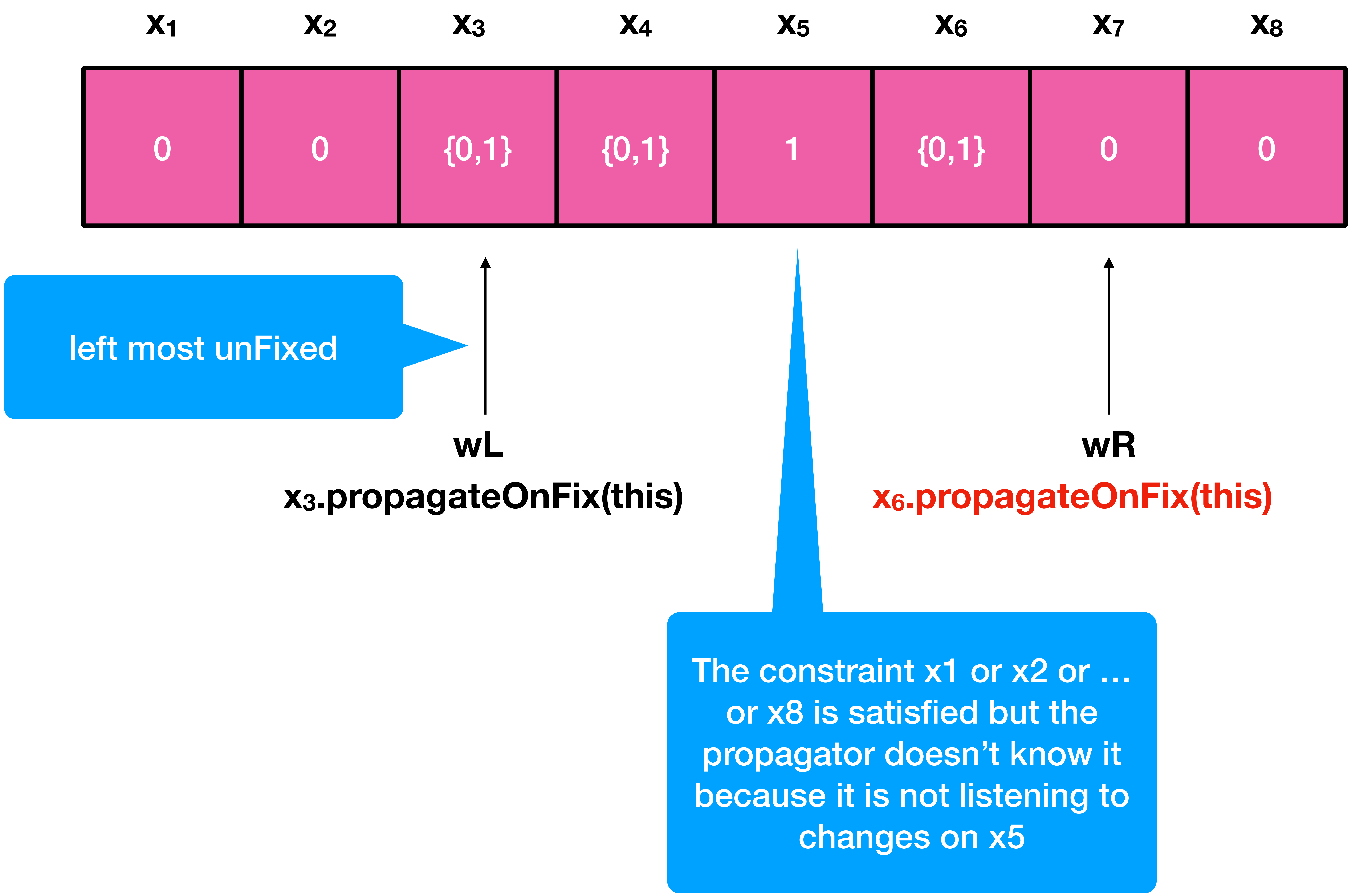

Unit Propagation

- If wL = wR (only one variable != 0), it must be set to 1
- If wL > wR, all variable are zero, we must fail (inconsistency).

Reifled Or: IsOr Constraint

- b iff (x_1 or x_2 or … or x_n)
  - b = true: post (x_1 or x_2 or … or x_n) = the Or constraint and deactivate
  - b = false: set all variables x_i to false
  - x_i become true: set b to true and deactivate (we must listen to all variables)
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- Exercise: bijection on graph coloring for value symmetries
- Redundant constraints
- Static vs Dynamic symmetry breaking